Global Nature of Dilute-to-Dense Transition of Granular Flows in a 2D Channel

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The dilute-to-dense transition of granular flow of particle size $d_0$ is studied experimentally in a two-dimensional channel (width $D$) with confined exit (width $d$). Our results show that with fixed $d$ and $D$ there is a maximum inflow rate $Q_c$ above which the flow changes from dilute to dense and the outflow rate ($Q(t)$) drops abruptly from $Q_c$ to a dense rate $Q_d$. A rescaled critical rate $q_c$ is found to be a function of a scaling variable $\lambda$ only: $q_c \sim F(\lambda)$, where $\lambda = \frac{d_0}{D}$. This form of $\lambda$ suggests that the dilute-to-dense transition is a global property of the flow, unlike the jamming transition which depends only on $\frac{d}{D}$. Furthermore, the transition is found to occur when the area fraction of particles near the exit exceeds a critical value which is close to 0.65 ± 0.03.

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The phenomena of crowding or jamming are common experiences in our daily lives. However, advances in the understanding of these processes have only begun recently after the physics of granular materials have been under intense investigations [1,2]. It is now believed that the diverse phenomena of traffic flow, pedestrian flow, and floating ice [3–5] are related to the nonlinear behaviors of granular materials which can exhibit both solidlike and fluidlike behaviors [6–8]. These peculiar properties [8,9] give rise to at least three important “states” in granular flows, namely, the dilute flow, dense flow, and the jammed state. The phenomenon of crowding can then be understood as a transition from dilute to dense flows and that of jamming is a transition from dense flows to a jammed phase.

Obviously, the nature and properties of these transitions are governed by the interactions among the granular particles in the flow. In principle, these transitions and states can be well characterized when these interactions are known such as in the cases of equilibrium systems [10,11]. Unfortunately, since interactions among the granular particles are highly nonlinear and characterized by dissipations which can be density and even history dependent, it is not surprising to find that these transitions and their states are still not well understood. Recently, there has been some progress in the understanding of the transition from dense to jammed states [12,13] and even an equilibriumlike jamming phase diagram has been proposed [14–16]. However, still very little is known about the dilute to density flow transition.

An important characteristic of the dense-to-jam transition is that there is usually only one dominant length scale in the problem, namely, the size of the grains ($d_0$) [12]. In contrast, it seems that the global scale of the order of the size of the system is important for the dilute-to-dense transitions. For example, a small bottleneck of the size of the system can induce the dilute-to-dense transition [17]. In this aspect, the dilute-to-dense transition is similar to transitions in hydrodynamic flows. In the case of hydrodynamic flows, it is well known that different Reynolds numbers are associated with different flow configurations to characterize the flow as laminar or turbulent. The Reynolds number is a global parameter which scales with the system sizes. Intuitively, in the case of dilute-to-dense transitions, a similar global scaling parameter might exist. If such a scaling parameter can be found, its scaling form will probably provide a better understanding of the nature of the dilute-to-dense transitions.

In this Letter, we report our results on experiments carried out in a 2D channel to look for a relation between the dilute-to-dense transition and system parameters. We find that the critical flow rate $Q_c$ at the transition can be well characterized by the scaling variable $\lambda = \frac{d_0}{D}$ where the system parameters are the channel width ($D$), the opening of the channel ($d$) and the diameter of the grains ($d_0$). $Q_c$ is the maximum dilute flow rate above which the flow changes from dilute to dense. The scaled critical flow rate $q_c (\equiv Q_c/(D/d_0))$ is found to be a function of a scaling variable $\lambda$ only as $q_c \sim F(\lambda)$. This form of $\lambda$ suggests that the dilute-to-dense transition is a global property of the flow, unlike the jamming transition which depends only on $\frac{d}{D}$. Furthermore, the transition is found to occur when the area fraction of particles near the exit exceeds a critical value which is close to 0.65 ± 0.03.

Our experiments are performed in a two-dimensional (2D) channel with an inclination angle of 20°. The 2D channel is established on a metal plate between two glass plates separated by specially shaped metal spacers to form a test section and a hopper as shown in Fig. 1. The gap between the two glass surfaces is kept at 1.2 mm (2.2 mm) to ensure an almost single-layer flow of steel beads of diameter $d_0 = 1 \pm 0.01$ mm (2 ± 0.01 mm) which are stored in the hopper at the top of the channel.
The hopper, with an open angle of 60°, is connected to a test section of width $D = 24$ mm and length $l = 500$ mm. At the end of the test section, there is an exit of width $d$ which is controlled by micrometers to a precision of 0.01 mm. Granular flows in the test section are initiated by allowing the steel beads in the hopper to fall by gravity. A thin plate with a number of uniformly distributed holes is inserted close to the exit of the hopper to control the inflow rate and ensure the uniformity of particle distribution across the test section. The total mass $M$ of the beads falling out of the exit is measured as a function of time $t$ by an electronic balance with sensitivity of 0.02 g and a weighing period of 0.02 s. The flow rate $Q(t)$ is obtained by the slope of the recorded $M(t)$ curve.

The flow in the test section initiated from the hopper is dilute and accelerating. The typical velocity of the steel beads close to the exit is $1.0 \pm 0.1$ m/s. Since $d$ is smaller than $D$, two wedges (heaps) will be formed at both sides of the exit with a base length $(D - d)/2$. If the inflow rate $Q_0$ is small or $d$ is not too small, there will be no net accumulation of beads in the test section other than the two wedges and $\langle Q(t) \rangle = Q_0$. This is the regime of dilute flows as shown in (a) of Fig. 1(B). However, for a given $Q_0$, there will be a critical $d_c$ below which there will be net accumulation of beads in the test section. In other words, if $d$ is decreased systematically, there will be a sudden drop of $Q(t)$ when $d_c$ is reached. This accumulation of beads will proceed until the whole test section is filled with beads and this is the regime of dense flows. The process of this dilute-to-dense transition induced by reducing $d$ is shown in Fig. 1(B), which is recorded by a video camera.

Figure 2 shows the $d$ dependence of $\langle Q(t) \rangle$ when $d$ is reduced or increased systematically for a given $Q_0$. When $d$ is large (point A), $\langle Q(t) \rangle = Q_0$, the flow is dilute. It can be seen that $\langle Q(t) \rangle$ remains practically independent of $d$ when $d$ is larger than a critical size $d_c$ (A to B in Fig. 2). When $d_c$ is reached, the dilute flow turns to dense flow and the flow rate can be reduced instantaneously by several times to drop from $Q_0$ to $Q_d$ (B to C). After the transition, $Q_d$ decreases monotonically with reducing $d$ (C to D). The flow jams when $d$ is about the size of four-particle diameters, where permanent arching occurs to cause jamming of the flow [12] (D to E). When increasing $d$ from the jammed phase, the flow starts as dense flow, and the rate $\langle Q(t) \rangle$ increases gradually with increasing $d$ until $Q_d$ reaches $Q_0$ and turns back to dilute flow as shown by triangle points in Fig. 2 (D through C to A). There is no sudden increase of the flow rate at the transition from dense to dilute flow. We have checked that the dense flow rate curve $Q_d(d)$ (the CD part of curve ACD) follows the Beverloo empirical equation $(d - kd_0)^{3/2}$ with $k = 4$ [18,19] and is independent of $Q_0$.

If the experiment of Fig. 2 is repeated with different $Q_0$, a family of paths similar to ABCD of Fig. 2 will be formed, shown as dotted lines in Fig. 2. An important characteristic of this family of lines is that $d_c$ decreases with $Q_0$ shown as broken line BF. That is, for a given $d$, there will be a $Q_0$ [denoted as $Q_c(d)$] at which a dilute-to-dense transition will occur. Figure 2 is the result of experiments with fixed $D$ but obviously $Q_c(d)$ will also depend on $D$ and $d_0$. The $Q_c(d)$ curves for various $D$ and $d_0$ have been measured and shown in Fig. 3. Four $D$’s, $D = 30, 25, 20,$ and $15$ mm, are tested for $d_0 = 1$ mm.

![FIG. 1. (A) Top and side views of the inclined channel. (B) Photos of dilute-to-dense flow transition in a time sequence](image-url)
particles, and two D’s, D = 40 and 30 mm, are tested for 2 mm particles. In Fig. 3 the upper six curves are \( Q_d(d) \)’s and the lower six curves are the corresponding \( Q_d(d) \) curves for particles \( d_0 = 1 \) and 2 mm. If the dilute-to-dense transition scales with the system size, one might expect that the exit width \( d \) should be scaled by channel width \( D \) as \( Q_d(d)/D = F(d/D) \) for some scaling function \( F \). The inset of Fig. 4 shows such a plot and obviously such scaling does not hold. Instead, a new scaling variable \( \lambda \equiv (d/d_0)(D/d_0) \) is found to collapse the scaled critical flow rate \( q_c \) \( \equiv Q_c(d)/(D/d_0) \) into a single scaling curve as shown in Fig. 4. The result in Fig. 4 can be expressed as \( q_c = q_0 F(\lambda) \), where \( q_0 \) is some constant. It means that the maximum flow rate of a dilute flow of a system of \( d \) and \( D \) is not the same as that of a system of 2d and 2D although their aspect ratio is the same.

A remarkable feature of the dilute-to-dense transition is that there are strong fluctuations in \( Q_t(t) \) when \( d \) is set to close to \( d_0 \). Direct observations of the motions of the beads close to the exit reveal that there are avalanchelike events taking place at the two wedges on either side of the exit. After the wedges have been built up by the incoming flux, flows of surface layer in the form of avalanches will occur. The two wedges effectively act as collectors of incoming flux and direct them to the exit through the surface flow. Therefore, there are strong density fluctuations (in terms of area fraction \( p \)) near the exit. When the discharge from the two wedges meet at the exit, the outflow can sometimes be blocked, leading to dense flow. This blockage is intermittent if the incoming flux is not large enough and therefore producing strong fluctuations in \( Q_t(t) \). However, if the incoming flux exceeds \( Q_c \), the blockage becomes permanent and there is accumulation of the beads in the test section until the whole section is filled. Particle density \( \rho \) close to the exit can be determined from video pictures. It is found that a dilute-to-dense transition will occur if \( \rho \) exceeds a critical value \( \rho_c \). In our experiments, it seems that the value of \( \rho_c \) always corresponds to that of an area fraction of 0.65 ± 0.03 and is independent of \( Q_d, d, \) or \( D \).

Following the observations discussed above, the flow across the test section can be divided into three regions: the central part with length \( d \) and the two wedges with base length \( D - d \). If \( v_0 \) and \( \rho_0 \) are the velocity and density of the dilute flow initiated by the hopper just before particles reaching the wedges, the incoming flux on the wedge is \( v_0 \rho_0 D - d \). The outgoing flux of the wedge will be carried away by the fluidized surface layer mentioned as \( v_r \rho_r \delta \), where \( \delta \) and \( v_r \) are the depth and characteristic velocity of the surface layer, respectively. Conservation of flux gives \( \delta = \frac{v_0 \rho_0 D - d}{v_r \rho_r} \). In this model, \( \delta \) will increase with the incoming flux or \( \rho_0 \). When these two top layers meet at the exit, we have \( \delta \beta = d/2 \), where \( \beta \) is some geometric factor which takes care of the angle of repose. Therefore, at the transition, we have \( f_c \equiv (v_0 \rho_0) = v_r \rho_r \beta^{-1} \frac{d}{D - d} \) or \( f_c = \frac{v_r}{v} \rho_r \beta^{-1} \), where \( f^* = v^* \rho_r \beta^{-1} \). One can consider \( v^* \) as some intrinsic velocity of the problem which is determined by physical properties of the system such as the angle of inclination or coefficient of sliding friction, etc. Presumably \( v^* \) is a function of system parameters \( D \) and \( d \). However, for the case of \( D \gg d \), it is reasonable to assume that \( v_r^* \) depends only on \( \frac{d}{d_0} \). In such a case, one expects to see \( \frac{v_r}{v^*} \sim \frac{d}{d_0} \) because this is the first order expansion of \( v_r \) in terms of \( d \) when \( d = 0 \) gives \( v_r = 0 \). This later form of \( v^* \) gives \( \frac{v_r}{v^*} \sim \lambda \equiv \frac{d}{d_0} \), which agrees with our result in Fig. 4 for small \( d \) or \( \lambda \).

Physically \( v^*(v^* \sim d^\alpha/d_0) \) means that the discharge velocity of the fluidized layer of the two wedges increases with \( d \). Note that \( v_r \) for the dense flow, \( v_r \sim d^{\alpha/2} \), with \( \alpha = 1/2 \) [19].

It can be seen from Fig. 4 that \( q_c \) increases with \( \lambda \) monotonically. Obviously, for a fixed \( D, Q_c \), must tend to a

![FIG. 3. The \( Q_d \) and \( Q_t \) vs \( d \) of particle size \( d_0 = 1 \) mm at channel widths \( D = 30, 25, 20, \) and 15 mm, and of particle size \( d_0 = 2 \) mm at channel widths \( D = 40 \) and 30 mm.](image)

![FIG. 4. The rescaled \( q_c \) (\( Q_c = Q_c/(D/d_0) \)) vs a new scaling variable \( \lambda = (d/d_0)(D/(D-d)) \), while \( Q_c/D \) vs \( d/D \) are shown in the inset (\( d_0 = 1 \) mm in this case). The dotted line is a fit of \( q_c \) by \( q_m(1 - e^{-\lambda/\lambda_0}) \), where \( q_m = 216 \) and \( \lambda_0 = 45 \).](image)
limit for large enough $d/d_0$ in the experiments because $Q$ is given by $v_0 \rho_0 D$ and there must be an upper limit in $\rho_0$ to still have dilute flows in the test section. If $q_m$ is the maximum dilute flow flux of the channel, we have $q_m = q_0 F(\infty)$. One can choose $F(0) = 0$ and $F(\infty) = 1$ to give $q_c = q_m F(\lambda)$. Note that there are two ways for $\lambda$ to go to $\infty$. When $d = D$, the test section is just a straight pipe (2D), $q_m$ is obviously just the maximum dilute flow capacity of the pipe. For fixed $d/D$, $\lambda$ goes to $\infty$ when either $d$ goes to $\infty$ or $d_0$ goes to zero. In both cases, we have a continuum limit in which the size of the particles can be neglected. Therefore, our model predicts that $q_m$ is independent of $d/D$ when $d/d_0$ is large enough. The functional form of $F(\lambda) = 1 - e^{-\lambda/\lambda_0}$, which gives the correct form of $q_c$ for small $\lambda$, has been fitted to the data in Fig. 4 shown as a dotted line. The scattering of the measured values of $Q_c$ increases as $\lambda$ increases, which makes it difficult to determine experimentally the limit value of $q_c$. Presumably, our model discussed above is valid only for small $d$.

The phenomenological model described above is based on the observation that the transition occurs when the flow density near the exit reaches $p = 0.65 \pm 0.03$. The abrupt change of the flow rate at the dilute-dense transition may be understood as the inelastic collisions of the particles with the two piles near exit. In some aspects, our system is similar to the wedge setup of Rericha et al. [20], where shocks identical to those in a supersonic gas are observed when a steady flow passes the wedge. Our system, however, with the piled heaps at the two sidewalls near the exit is equivalent to a system where particles pass two inward "soft wedges," a system more commonly seen in industrial transport of granules. The abrupt change in density, granular temperature, and velocity before and after the dense area near the exit may account for the flow rate drops at the dilute-dense transition, as the flow rate is a function of the product of flow density and the particle velocity. A two-dimensional molecular dynamics simulation using force model similar to [21] is performed to provide the microscopic view of the transition process. Simulation results show that the dense flow and the dilute flow are two different states. When the flow density reaches $p = 0.65 \pm 0.03$, the flow becomes dense flow and the multibody collisions dominate.

In conclusion, we have experimentally obtained a dilute-to-dense transition curve $Q_c(Q_0, d_0)$, at which the flow rate $Q_0$ drops to $Q_0$ at $d_c$. Instead of the intuitive scaling relationship $d/D$, experimental results show that a global scaling variable $\lambda = (d/D - d)(d/d_0)$ determines if the flow is dilute or dense at a given $Q_0$. The scaled $q_c$ can be fitted in an empirical form $q_m(1 - e^{-\lambda/\lambda_0})$. The ratio $\lambda/\lambda_0$ determines the critical flux of the system. Intuitively, $\lambda_0$ is determined by physical properties of the system, such as the elasticity of the particles, particle size, and the inclination angle of the plate, etc. While the physics of jamming transition is determined by local scales close to $d_0$, our result suggests that the physics of dilute-to-dense transition is similar to hydrodynamic instability which is controlled by global scales such as $d/(D - d)$. The discovery of this global scaling property may provide us with ideas of better designs for the transport of granules in industrial processing, and better understanding of similar flow systems such as systems in traffic flows.

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