Quantum Magnetic Dynamics of Polarized Light in Arrays of Microcavities

An-Chun Ji,1 X. C. Xie,2,1 and W. M. Liu1

1Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100080, China
2Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078, USA
(Received 20 June 2007; published 1 November 2007)

We describe an optical system that allows for a direct experimental observation of the quantum magnetic correlated dynamics of polarized light. By adjusting the Zeeman and the Raman fields, we could realize a ferromagnetic phase, super-counter-fluidity phase, and antiferromagnetic phase of polarized light, that are of interest for studying spin-dependent photon-photon interactions. We also design an experimental protocol for the observation of these phases. Moreover, the technique of controlling photospin correlation may be used for building quantum information devices.

DOI: 10.1103/PhysRevLett.99.183602 PACS numbers: 42.50.Pq, 03.67.-a, 03.75.Nt, 05.30.Jp

The observation of effective photonic repulsion [1] has triggered exciting possibilities for exploring strongly correlated dynamics of an optical system [2–4]. For example, it is now possible to experimentally study the Mott insulator-superfluid quantum phase transition of photons [2,3], and to simulate the dynamics of the XY spin chains [4]. Compared to other strongly correlated many particle systems in condensed matter physics or cold atoms, an optical system has the advantage of accessing to individual lattices, and it does not require nano-Kelvin temperatures. Thus, it offers the ability to experimentally observe quantum-many-body phenomena and design quantum-mechanical devices for quantum information processes.

In this Letter, we explore spin-dependent photon-photon correlation effects of polarized light. Very recently, the magnetic effects of polarized light have been considered: Onoda et al. considered the polarization-dependent Hall effect of light [5], and Jonsson et al. considered photospin-orbit coupling in photonic structures [6]. Experimentally, Wilk produced a stream of single photons with alternating circular polarizations [7], and García-Maraver et al. achieved a deterministic generation of polarization entangled photons pairs [8].

In this work we setup an optical system consisting of coupled polarization-degenerate microcavities with a single V-type three-level atom within each cavity. We are interested in the Mott regime where there is essentially one photon per cavity. This technique, for the first time, provides a practical way to explore the quantum magnetic correlation of polarized photons. By adjusting the Zeeman or the Raman fields, the ground state of the system may undergo a ferromagnetic phase, an antiferromagnetic phase, or a super-counter-fluidity (SCF) phase, that breaks the easy-plane U(1) symmetry. The SCF phase was first predicted by Kuklov [9] for two-species ultracold atoms in a commensurate optical lattice. Here we illustrate how to implement this phase in an optical system and its experimental detection. Furthermore, by controlling the spin correlation of the photons, one might obtain photospin ladder, that could be useful in the generation of the self-ordered array of emitters with polarization entangled photon pairs.

The system under investigation is schematically depicted in Fig. 1(a). We have chosen a compressed 2D hexagonal photonic crystal, where the lattice compression is used to modify the in-plane bandstructure, and modify the dominant Fourier components found in the defect...
modes [10]. Moreover, we have chosen the fundamental even (TE-like) modes with the magnetic field perpendicular to the slab. Figure 1(b) shows the Brillouin zone of the compressed hexagonal lattice, whose point group symmetry is reduced from \( C_{6v} \) to \( C_{2v} \) by compression. The evanescent coupling between cavities drops off exponentially with distance, and we therefore consider nearest-neighbor photon hopping. To achieve the regime where photonic repulsion dominates over hopping, we have chosen four nearest neighbors per cavity (coordination number \( z = 4 \)). The finite-difference time-domain (FDTD) method showed that this configuration could yield \( Q \) value in excess of \( 10^5 \) with mode volume of approximately 0.35 cubic half-wavelengths in vacuum.

To affect the photon-photon spin exchange interactions, we insert a single \( V \)-type three-level atom into each cavity, which can be realized using single-ion implantation techniques [11]. Figure 1(c) shows the relevant levels involved, the \( J = 0 \) ground state, \( | g \rangle \), and the \( J = 1 \) excited states, labeled by magnetic sublevel \( | c \rangle = |-1 \rangle \), \( | d \rangle = |+1 \rangle \), and \( | 0 \rangle \). Each atom has two electric dipole transition energies \( \epsilon_c \) and \( \epsilon_d \). We further include a spatially dependent effective magnetic field \( B \) to vary coupling constants, with the \( B \) field perpendicular to the two-dimensional photon crystal slab. The \( B \) field defines the quantization axis of photospin and lifts the degeneracy of the excited states, producing an energy shift of the \( m = \pm 1 \) states of \( \mp \Delta_{Bi} \). The cavities support only \( \sigma^+ \) and \( \sigma^- \) photons with the identical mode resonance frequency \( \omega \), and the cavity-mediated atom-photonic coupling is denoted as \( \beta \). We introduce the detunings, \( \Delta_i^+ = \Delta - \Delta_{Bi} \), and \( \Delta_i^- = \Delta + \Delta_{Bi} \), where \( \Delta \) is the zero field detuning.

The Hamiltonian for the photospin interaction system is described by the spin-dependent Janes-Cummings (JC) model [12], combined with photon hopping between cavities. In the rotating-wave approximation, the many-body dynamics of the full system is given by the following Hamiltonian

\[
\hat{H} = \hat{T} + \hat{V} = -\sum_{i,j} \sum_{\sigma=\pm} \kappa_i^\sigma \hat{a}_i^\dagger \hat{a}_j^\sigma + \text{H.c.} + \sum_i \hat{H}_i^{\text{JC}},
\]

\[
\hat{H}_i^{\text{JC}} = \sum_{\sigma=\pm} \kappa_i^\sigma \hat{a}_i^\dagger \hat{A}_{i;i}^\sigma \hat{a}_i^\sigma + \kappa_i^\sigma \hat{a}_i^\dagger \hat{A}_{i;i}^\sigma \hat{a}_i^\sigma + \beta \hat{A}_{i;i}^\sigma \hat{A}_{i;i}^\dagger + \hat{A}_{i;i}^\sigma \hat{A}_{i;i}^\dagger + \hat{A}_{i;i}^\dagger \hat{A}_{i;i}^\dagger,
\]

where \( \hat{a}_i^\dagger (\hat{a}_i) \) is the creation (annihilation) operator of the left (right) circularly polarized photons or the photospin up (down) states in each cavity mode. \( \hat{A}_{i;i}^\sigma \) and \( \hat{A}_{i;i}^\dagger \) correspond to the \( m = \pm 1 \) state raising and lowering operators of each atom. \( \kappa_i^\sigma = \kappa \) for the nearest neighbors photon hopping, and \( \kappa_i^\sigma = 0 \) otherwise.

We shall first solve the photospin Janes-Cummings Hamiltonian, then derive an effective exchange interaction model, and then give a detailed description of the experimental signatures of these many-body phases. Though we are considering one photon per cavity, the system can exist in excited states where the cavities are multiply occupied. We confine ourselves to the regime where these energies are large compared with the photon hopping energy \( \kappa \).

The eigenstates of Eq. (1) are the dressed states [13], which we define as \( \{ \pm, \sigma \} \), for one photon with spin \( \sigma \) per cavity and \( \{ \pm, \sigma \sigma' \} \), for two photons with spin \( \sigma \) and \( \sigma' \) per cavity.

We first consider the one-photon ground state. The normalized eigenstates are

\[
|+, \sigma \rangle_i = \sin \theta |g, \sigma \rangle - \cos \theta |\lambda, 0 \rangle,
\]

\[
|-, \sigma \rangle_i = \cos \theta |g, \sigma \rangle + \sin \theta |\lambda, 0 \rangle,
\]

where \( |\lambda \rangle \) is the corresponding atom excited state for \( |\sigma \rangle \) state photon. The eigenenergies are \( E_{\{\pm, \sigma\}} = \omega - \Delta_i^\sigma / 2 \mp \chi_i(\sigma) \), where \( \chi_i(\sigma) = \sqrt{\beta^2 + (\Delta_i^\sigma)^2} / 4 \). Note that \( E_{\{+, \sigma\}} < E_{\{+, +\}} \), we only need to consider the negative branch as the true one-photon ground state [2].

Next, we consider two-photon excited states. The eigenenergies have two branches too, and we choose the negative branch. It is easy to determine the eigenvalue of a cavity with two photons of the same spin. \( E_{\{-, \sigma \sigma\}} = 2 \omega - \Delta_i^\sigma / 2 - \chi_i(\sigma \sigma) \), where \( \chi_i(\sigma \sigma) = \sqrt{2 \beta^2 + (\Delta_i^\sigma)^2} / 4 \). To solve the excited state \( |-, |1 \rangle \), for a cavity containing two photons with opposite spin state \( || \rangle \) and \( |1 \rangle \), we write the normalized eigenstate as

\[
|-, |1 \rangle_i = \cos \theta |g, |1 \rangle + \sin \theta (\cos \phi |a, | \rangle + \sin \phi |b, | \rangle),
\]

where \( \theta \) is the mixing angle, and \( \phi \) is the asymmetry angle, which can be written as \( \phi = \pi / 4 + \alpha \). When the Zeeman field is turned off, \( \alpha = 0 \), the two excited states become degenerate, and they form a superposition state \( |c, | \rangle + |d, | \rangle \). This is the so-called bright state [14], and it mixes with the state \( |g, | \rangle \), giving the Rabi resonance. The two angles \( \theta \) and \( \alpha \) are determined by following equations

\[
2\Delta = \beta \cot \theta \left( \frac{1}{\cos \alpha} + \frac{1}{\sin \alpha} \right) - 2\beta \tan \theta (\cos \alpha + \sin \alpha),
\]

\[
2\Delta_B = \beta \cot \theta \left( \frac{1}{\cos \alpha} - \frac{1}{\sin \alpha} \right),
\]

and the corresponding eigenenergy is \( E_{|-, 0 \rangle} = 2\omega - \frac{\beta}{2} - \frac{\beta}{4} \tan \theta (\cos \alpha + \sin \alpha) - \frac{\beta}{4} \cot \theta (\frac{1}{\cos \alpha} + \frac{1}{\sin \alpha}) \).
In a uniform magnetic field, \( \Delta_B = \Delta_B \), the above Hamiltonian reduces to the anisotropic Heisenberg model with \( J, J^* < 0 \). The ground state of the system has ferromagnetic order, with all the photons right or left circularly polarized. However, it is possible to explore other phases by varying the effective coupling constants. We note that if \( J^* > 0 \), the ground state of this system is then the easy-plane ferromagnet, which supports the super-counter-fluidity. To explore this phase, we begin with a configuration of an small staggered field, \( \Delta_s = (1)^\ell \Delta_B \), small detuning, and strong coupling, i.e., \( \beta > \Delta, \Delta_B \). In these limits, we arrive at \( \frac{1}{\nu_{ll}} = \frac{1}{\nu_{ll}} = \frac{D}{D^2 - \Delta_B^2/4} \), and the corresponding coupling constants \( J^* = \frac{6 \Delta_s^2 \hbar c^2}{(D^2 - \Delta_B^2)(D^2 - \Delta_B^2/4)} \), here \( D = (2 - \sqrt{2}) \beta + \Delta/2 \). Then the easy-plane condition \( J^* > 0 \) can be realized if the staggered field \( \Delta_B < D \).

The super-counter-fluidity can be understood as follows. In an easy-plane anisotropic ferromagnetic system, the photospin raising operator can be written as \( \hat{S}_i^+ = \hat{a}_i^\dagger \hat{a}_i \), and the easy-plane U(1) symmetry could be broken only spontaneously by forming the superfluid counterflow vacuum, i.e., \( \langle \hat{S}_i^+ \rangle = \langle \hat{a}_i^\dagger \hat{a}_i \rangle \neq 0 \). This is closely analogous to exciton condensation [16], where photons and holes with different polarizations form pairs and condensate. This means that, while the transport of the net number of photons is still suppressed in the Mott regime, the currents of left or right circularly polarized photons are equal in absolute values and are in opposite directions.

Experimentally, we could construct a geometry to identify SCF, where two beam of opposite circularly polarized lasers are positioned in opposite directions for driving currents in and out of the system. To characterize the coherent phase formation, we consider the processes of interactions among photons and their detection. First we estimate the typical experimental time scales involved. The photon lifetime is determined by \( \tau_{ph} = Q/\omega_c \), for the transition wavelength 772 nm \( \sim 4 \times 10^{14} \) Hz, note that \( Q \sim 10^7 \) has been achieved [2], \( \tau_{ph} \approx 25 \) ns. After one turns on the staggered field, and the system will be in the SCF phase, the typical time of this coherent process \( \tau_{co} \) is determined by virtue process of two photons excitation, which is \( \sim 1/U \). For \( \beta \sim 10^{10} \) Hz, \( \tau_{co} \sim 0.1 \) ns, that is rather short compared to the photon lifetime. Then one feeds two directed laser beams to the sample, and \( Q \) switches the cavities [17], one could directly readout the number of two species of the polarized photons passing through the slab via a near-field probe with a polarizing beam splitter (PBS) [7,8]. It should be noted that, all these laser beams should be stable, with the stability \( < \min [\mu - \mu_c(0)], \mu - \mu_c(1)] \sim 10^{-4} \). Besides, there is no particular temperature requirement (see Ref. [2] and references therein) in this process. So this setup could serve as a good candidate for realizing optical switches.
To explore other phases, we further include effective electric fields, that can be engineered by using ac Stark shifts [18,19]. This technique was first used by García-Ripoll et al. [19] to implement the quadratic-biquadratic Hamiltonian for the spin $S = 1$ cold atoms in 1D optical lattice. By switching on two kinds of Raman lasers for two sublattices, one can produce the atom ground state energy shift $\delta_1$ for one sublattice, and $\delta_2$ for the other sublattice. In the absence of a magnetic field, the new eigenvalues of one-photon ground state and two-photon excited state are now given by $\hat{E}_{\pm 1}\left(\Delta\right)$ and $\hat{E}_{-2}\left(\Delta\right)$ with $\Delta = \Delta + \delta_1$, here $\delta_1$ is for one sublattice, and $\delta_2$ for the other one. Following the same procedure, we arrive at $J = \frac{4e^2(\Delta + \delta_1 + \delta_2)}{(\delta_1 - \delta_2)^2 - (\Delta + \delta_1 + \delta_2)^2}$, $J' = 0$ for the large detuning limit $|\Delta| > \beta$. We note that, if $\Delta + \delta_1, \Delta + \delta_2$ are of opposite sign, and $|\Delta + \delta_1| > |\Delta + \delta_2|$, then $J > 0$, signaling an antiferromagnetic coupling.

Experimentally, the antiferromagnetic ground state could be realized adiabatically with the help of the duality between ferro- and antiferromagnetic models $H_{AF} = -H_F$ [19]. To do that, one first constructs a configuration of antiparallel photospins by turning on an effective staggered magnetic field, then progressively decreases the magnetic field and increases the electric field gradient, and thus, arriving at the desired state adiabatically.

As an application, we consider photospin ladder with exchange energies $J_1$ along the legs and $J_R$ on rungs. This model is equivalent to the $S = 1/2$ Heisenberg ladder that is one of the simplest but most emblematic quantum-many-body systems. For all values of $J_R > 0$, the DMRG results by White [20] showed that the ladder is in the short-range resonating valence bond phase (dimer RVB) with singlets on each rung. The diagonally situated next-nearest-neighbor spins are coupled to form an effective $S = 1$, and this phase is identical to the Haldane phase. Thus, the photospin ladder could offer us a possible way to generate an array of self-ordered array of singlet state, which could be used as the emitters of polarization entangled photon pairs.

Until now, all the above analysis was focused on the schematically depicted photonic crystal slab. A possible implementation that may be realizable would be an array of micro-fabricated high-finesse optical cavities, with the cavities connected by optical fibers [21]. Each cavity is formed by a concave micromirror and the plane tip of an optical fiber, with open access and small mode volume, that could be designed to support only the TEM$_{00}$ mode. Single $V$-shaped atoms could be transported and held in each cavity by optical confinement and atom waveguide techniques [22–25]. We could choose an argon atom for our purpose, which has a metastable spinless ground state $1s_1$ ($J = 0$), and was successfully confined into a three-dimensional optical lattice by Müller-Seydlitz et al. [22]. The resonant excited level $2p_2$ ($J = 1$) is characterized by the spontaneous decay rate $\Gamma = 1.25 \times 10^7$ sec$^{-1}$, with wavelength 772 nm [26]. This system seems to approach the strong coupling regime, with $\beta \sim \kappa \sim 10^9$ Hz, and $Q$ possibly over $10^6$.

In summary, we have shown how to implement the spin correlation of polarized photons. Such experiments will allow us to observe directly the photospin magnetic correlation effects in each microcavity. We predicted that the system may posses the super-counter-fluidity in the ground state, and a mean to detect the SCF is present. Finally, we have investigated the photospin ladder, which may be beneficial for the realization of future quantum computation devices.

We thank C. J. Xu and F. Zhou for helpful discussions. This work was supported by NSFC under Grants No. 90406017 and No. 60525417, and the NKBRSFC under Grant No. 2006CB921400. X. C. Xie is supported by the US-DOE and NSF.